

## B.Sc. Semester - 5 (CBCS) Examination

Oct/Nov. - 2018

## BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours

Marks: 70

## Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q- Attempt any one out of two.

1(A) (1) Write and prove Distributive inequalities.

) (2)  $R = \{(x,y) / x,y \in \mathbb{Z} \text{ and } xy \text{ is divisible by } 4\}$ , then prove that  $R$  is equivalence relation. Also find all equivalence

Q- Attempt any one out of two.

1(B) (1) State and prove Modular inequalities.

(2) Let  $(L, *, \oplus, 0, 1)$  be a distributive lattice then prove that if complement element  $a \in L$  exist then it's a unique

Q- Attempt any one out of two.

1(C) (1) In usual notation prove that,  $xRy \Leftrightarrow [x] = [y]$ .(2) If  $n$  is a positive integer and  $S_n$  is a set of all factors of  $n$ ,  $a, b \in S_n$ .  $aDb$  means  $a$  divides  $b$ , then show that  $(S_n, D)$ 

Q- Attempt any one out of two.

2(A) (1) State and prove D' morgan's law for Boolean algebra.

) (2) State and prove stone's representation theorem.

Q- Attempt any one out of two.

2(B) (1) Let  $(B, *, \oplus, ', 0, 1)$  be a Boolean algebra and  $x_1, x_2 \in B$  then prove that  $A(x_1 * x_2) = A(x_1) \cap A(x_2)$ (2) Let  $(B, *, \oplus, ', 0, 1)$  be a Boolean algebra with  $\eta$  - variables  $x_1, x_2, x_3, \dots, x_n$  and  $m_i, m_j$  are two different m

Q- Attempt any one out of two.

2(C) (1) If  $(B, *, \oplus, ', 0, 1)$  is a finite Boolean algebra and  $x \in B$ ,  $x \neq 0$  then prove that there exist atom  $a$  in  $B$  such th(2) Obtain Pos canonical form of  $\alpha(x_1, x_2, x_3) = x_1 \oplus (x_2 * x_3)$ .

Q- Attempt any one out of two.

3(A) (1) Let  $f(z) = u(x,y) + i v(x,y)$  is a complex function and  $z_0 = x_0 + i y_0$  and  $w_0 = u_0 + i v_0$  are fixed complex constant a) 
$$\lim_{z \rightarrow z_0} f(z) = w_0 \text{ iff } \lim_{(x,y) \rightarrow (x_0, y_0)} u(x,y) = u_0$$

$$\text{and } \lim_{(x,y) \rightarrow (x_0, y_0)} v(x,y) = v_0$$

(2) State and prove Cauchy – Riemann conditions in Cartesian form.

Q- Attempt any one out of two.

3(B) (1) Show that  $\sinh x \sin y$  is harmonic function & find its conjugate harmonic function.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

Q- Attempt any one out of two.

3(C) (1) Show that  $z^2$  is entire function.

(2) Show that  $f(z) = e^y (\cos x + i \sin x)$  is not analytic function.

Q- Attempt any one out of two.

4(A) (1) State and prove Cauchy's Integral Formula.

) (2) State and prove Cauchy – Riemann condition for analytic function in polar form.

Q- Attempt any one out of two.

4(B) (1) In usual notation prove that,

$$\left| \int_a^b f(z) dz \right| \leq \int_a^b |f(z)| dz,$$

(2) P

Q- Attempt any one out of two.

4(C) (1) Find  $\int_C \frac{dz}{z^2+4}$ ;  $C : |z - i| = 2$ .

(2) In usual notation prove that

$$\left| \int_C f(z) dz \right| \leq ML.$$

Q- Attempt any one out of two.

5(A) (1) State and prove Morera's theorem.

) (2) State and prove the fundamental theorem of algebra.

Q- Attempt any one out of two.

5(B) (1) State and prove Cauchy's inequality.

(2) Evaluate:  $\int_C \frac{z^2+1}{(z-1)^3} dz$ , where  $C: |z| = 2$ .

Q- Attempt any one out of two.

5(C) (1) Evaluate:  $\int_C \frac{5z^2-7z+3}{(z-1)^2} dz$ . where  $C : |z| = 2$ .

(2) Evaluate:  $\int_C \frac{\cosh z}{z^3} dz$ . where  $C : |z| = 1$ .

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